

Teaching Physics at University by using « Open source » and « home-made » modelling tools

Hervé Bulou, IPCMS, Strasbourg
herve.bulou@ipcms.unistra.fr



« Simulation and Modelling of Complex Materials and Phenomena » Team

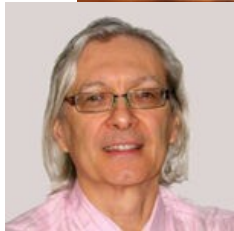


O. Bengone

H. Bulou

C. Goyhenex

T. Andriamiharintsoa



F. Maingot
de la
Grassière

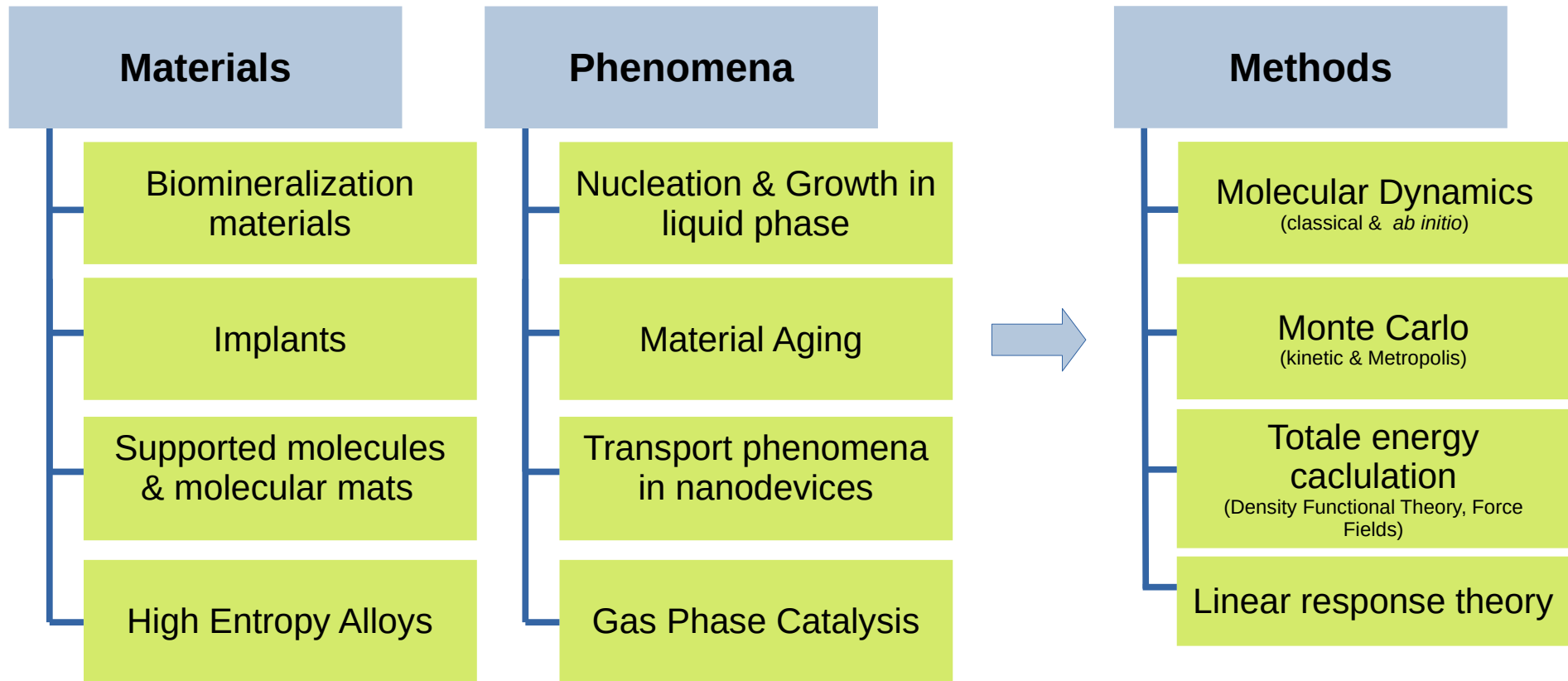
Permanent Members

- O. Bengone
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- F. Maingot de la Grassière

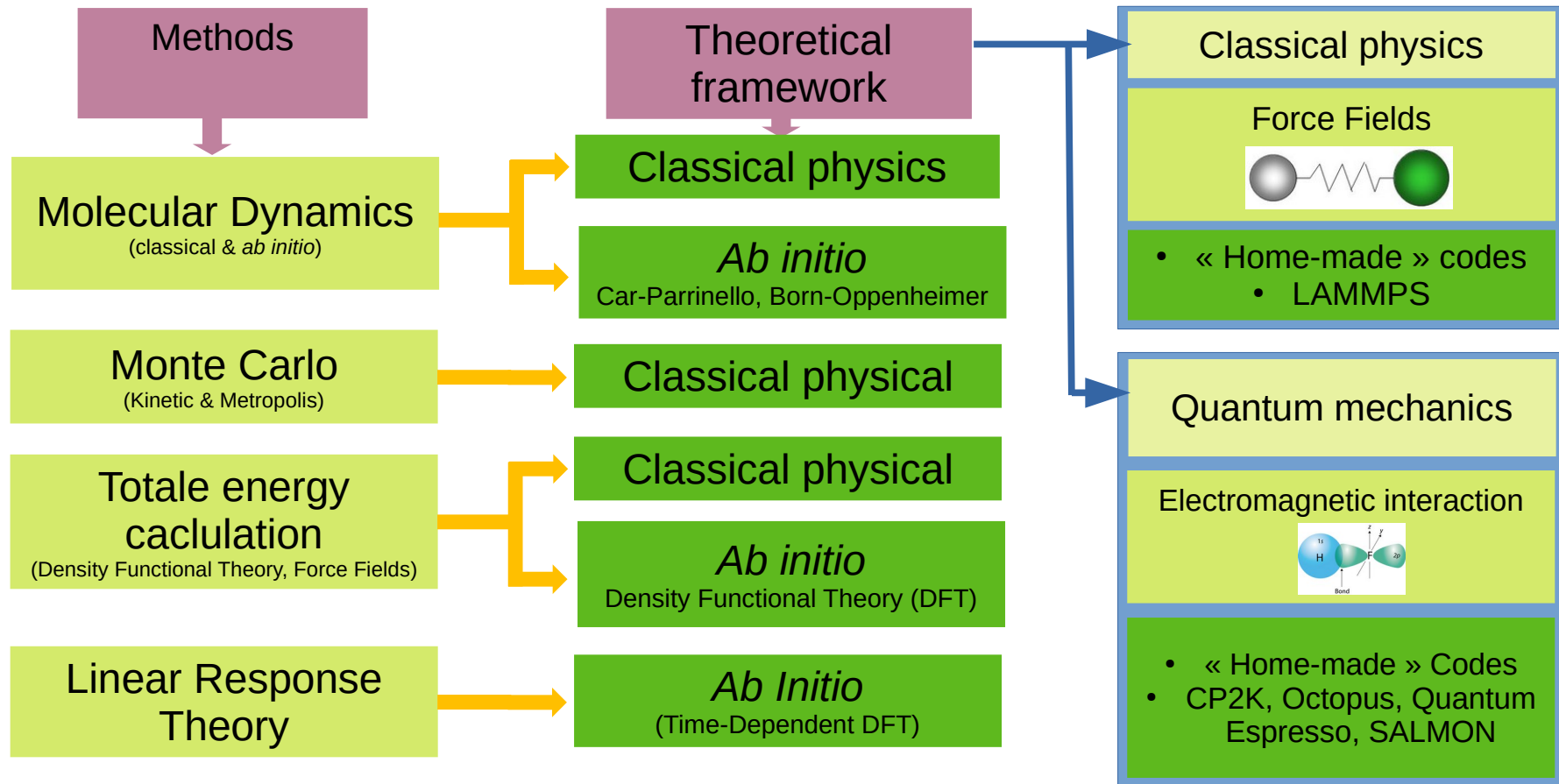


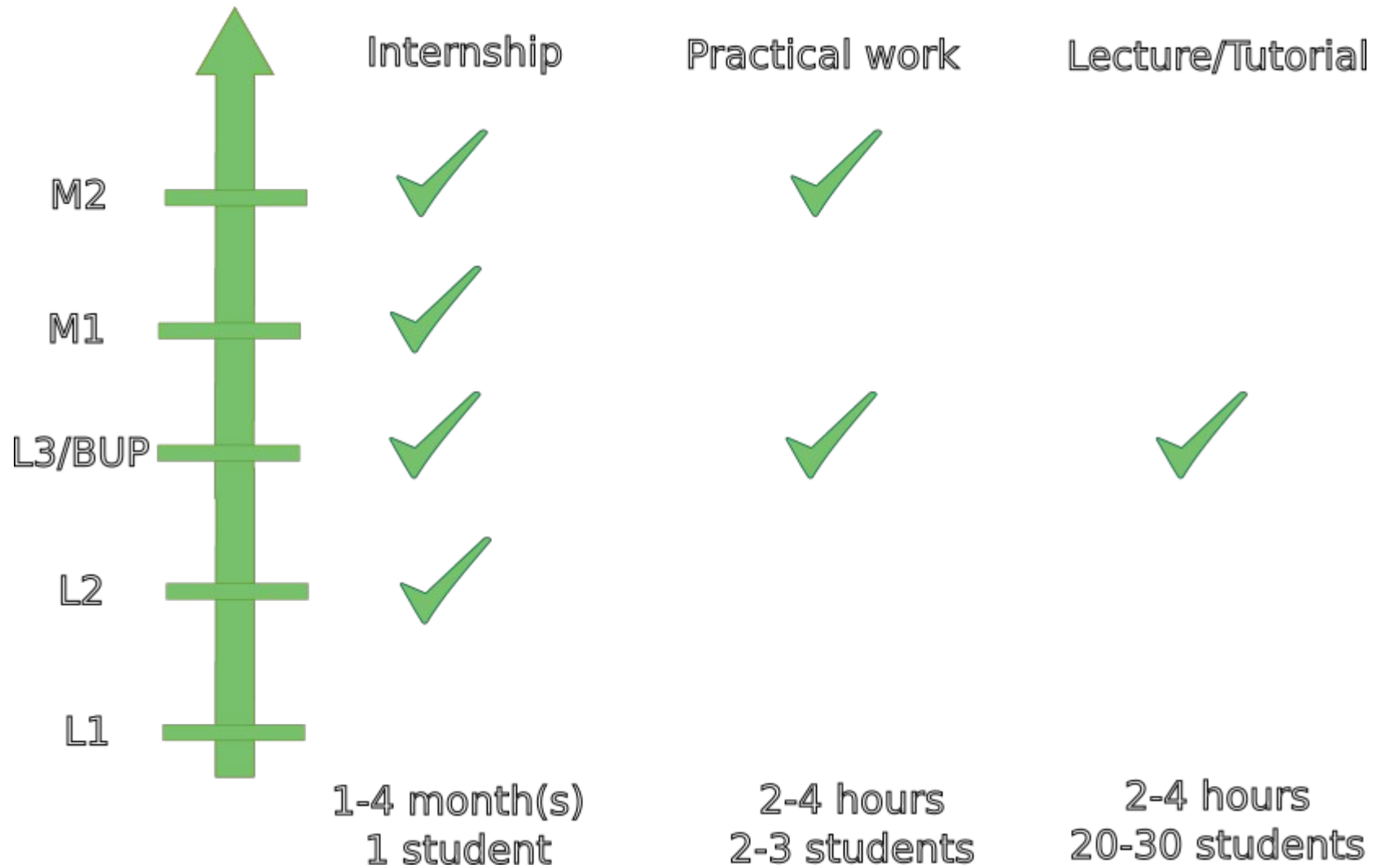
Institut de Physique et Chimie des
Matériaux de Strasbourg

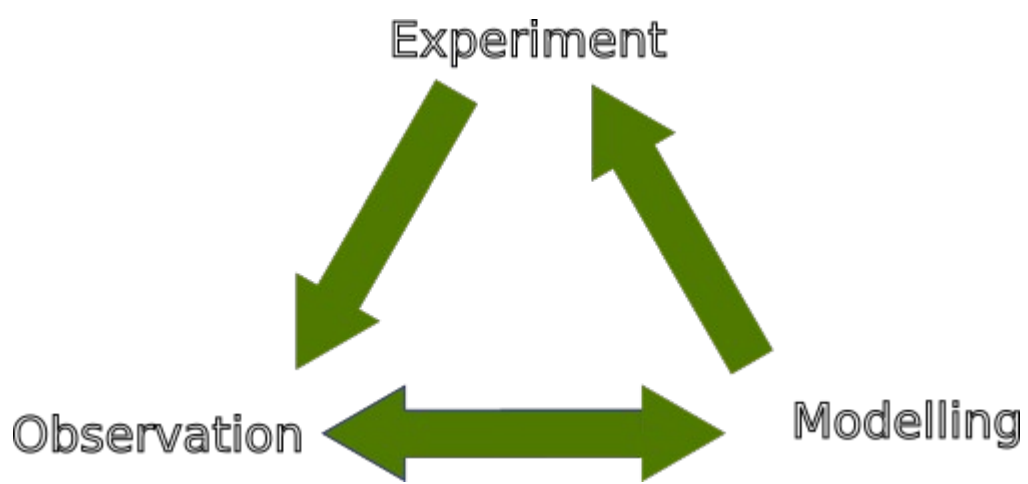
« Simulation and Modelling of Complex Materials and Phenomena» - SMCMP Team



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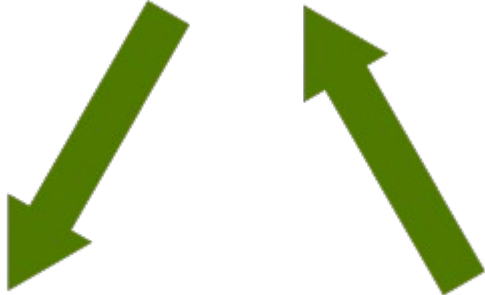






Experiment

Step 1: Qualitative description of the observation



Observation

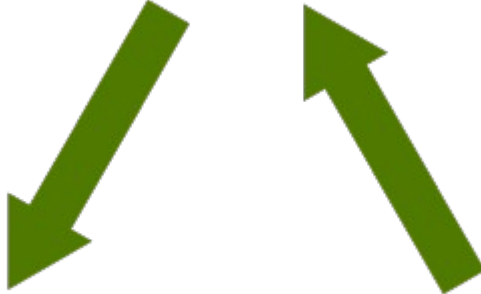


Modelling

Experiment

Step 1: Qualitative description of the observation

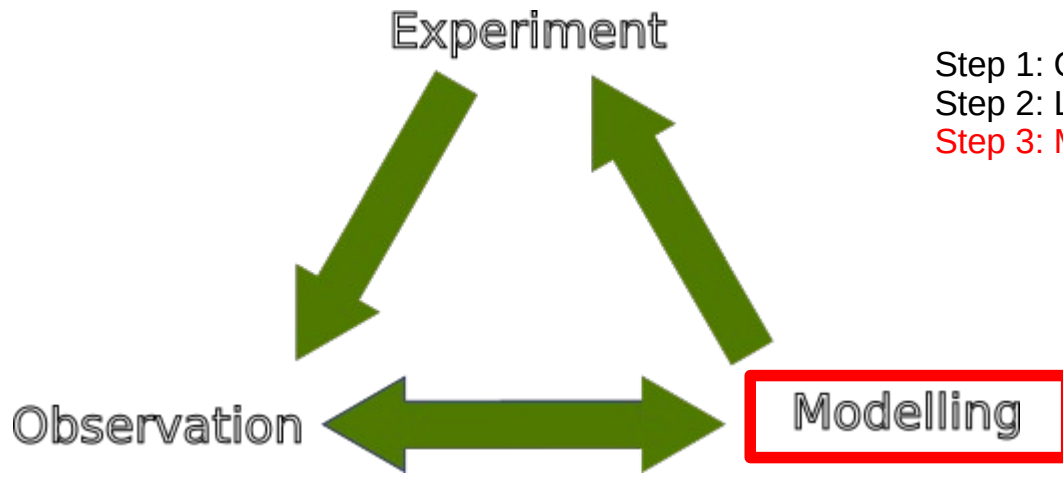
Step 2: List of basic phenomena involved



Observation



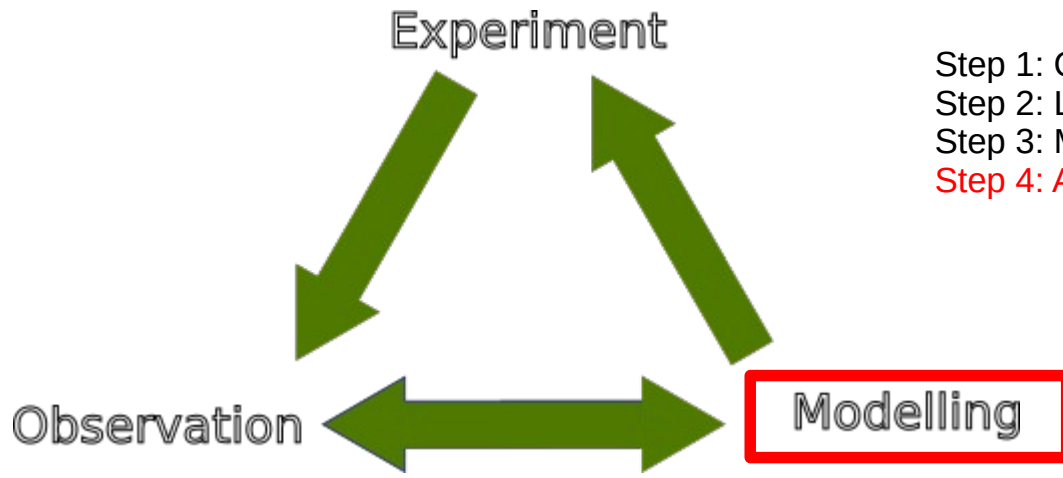
Modelling



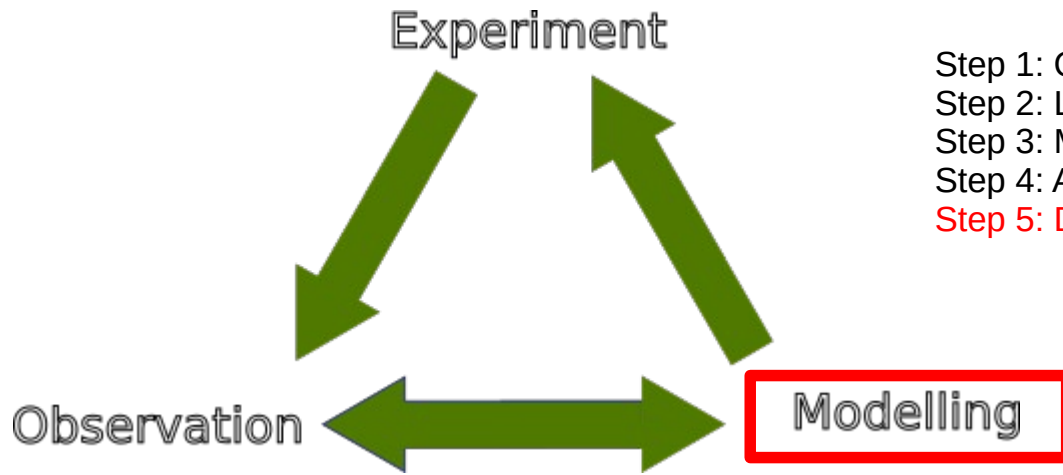
Step 1: Qualitative description of the observation

Step 2: List of basic phenomena involved

Step 3: Mathematical formulation of the observation

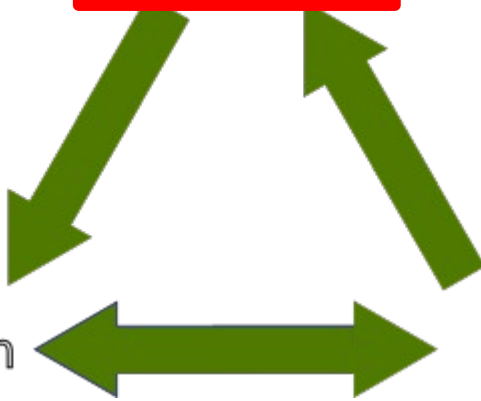


- Step 1: Qualitative description of the observation
- Step 2: List of basic phenomena involved
- Step 3: Mathematical formulation of the observation
- Step 4: Algorithmic formulation of observation



- Step 1: Qualitative description of the observation
- Step 2: List of basic phenomena involved
- Step 3: Mathematical formulation of the observation
- Step 4: Algorithmic formulation of observation
- Step 5: Development of code to reproduce the observation

Experiment



Observation

Modelling

Step 1: Qualitative description of the observation

Step 2: List of basic phenomena involved

Step 3: Mathematical formulation of the observation

Step 4: Algorithmic formulation of observation

Step 5: Development of code to reproduce the observation

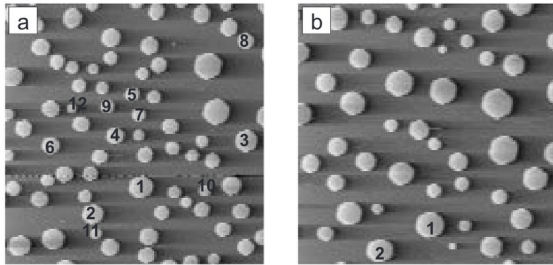
Step 6: Exploitation of the model to imagine new experiences

Experiment

- Step 1: Qualitative description of the observation
- Step 2: List of basic phenomena involved
- Step 3: Mathematical formulation of the observation
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- Step 5: Development of code to reproduce the observation
- Step 6: Exploitation of the model to imagine new experiences

Observation

Modelling



200 nm

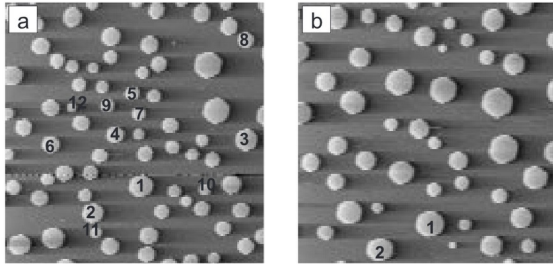
Fig. 1a,b. STM images taken from a movie sequence showing Ostwald ripening of 0.21 ML Ag on Ag(111) at room temperature. The left image (a) was recorded 38 minutes after the completion of deposition, the right image (b) shows the same sample another 6 h and 56 min later. The area of the islands marked by numbers on the first image is plotted in Fig. 2 as a function of time. For better orientation, the islands 1 and 2 are marked on the right image as well

Experiment

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Observation

Modelling

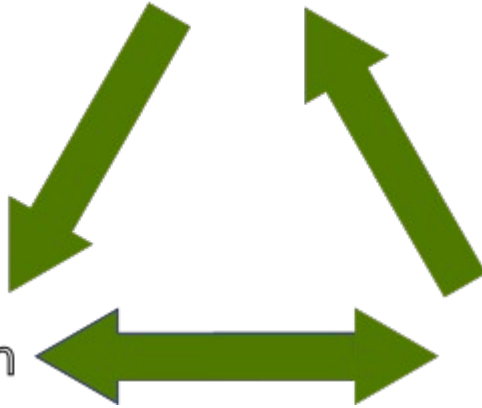


200 nm

Fig. 1a,b. STM images taken from a movie sequence showing Ostwald ripening of 0.21 ML Ag on Ag(111) at room temperature. The left image (a) was recorded 38 minutes after the completion of deposition, the right image (b) shows the same sample another 6 h and 56 min later. The area of the islands marked by numbers on the first image is plotted in Fig. 2 as a function of time. For better orientation, the islands 1 and 2 are marked on the right image as well

Mathematics: Taylor development $\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{d\mathbf{r}(t)}{dt}\delta t + \frac{1}{2} \frac{d^2\mathbf{r}(t)}{dt^2}\delta t^2 + \mathcal{O}(\delta t^3)$

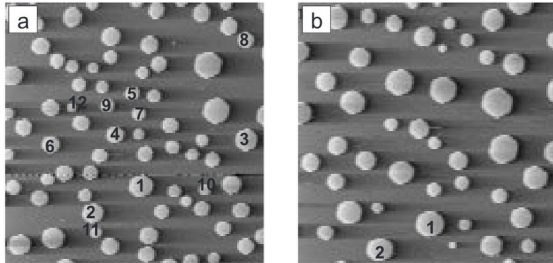
Experiment



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200 nm

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Mathematics: Taylor development

$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{d\mathbf{r}(t)}{dt}\delta t + \frac{1}{2} \frac{d^2\mathbf{r}(t)}{dt^2}\delta t^2 + \mathcal{O}(\delta t^3)$$

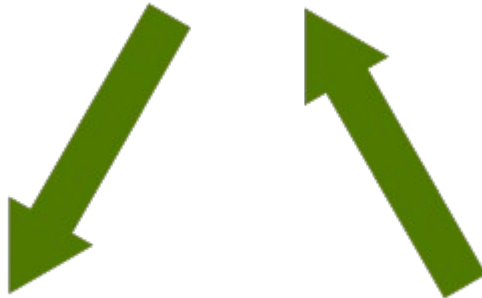


Physics: Newton equation

$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{\mathbf{p}(t)}{m}\delta t + \frac{1}{2} \frac{\mathbf{F}(t)}{m}\delta t^2 + \mathcal{O}(\delta t^3)$$

$$\mathbf{p}(t + \delta t) = \mathbf{p}(t) + \frac{1}{2m} (\mathbf{F}(t + \delta t) + \mathbf{F}(t)) \delta t + \mathcal{O}(\delta t^3)$$

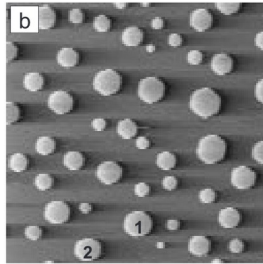
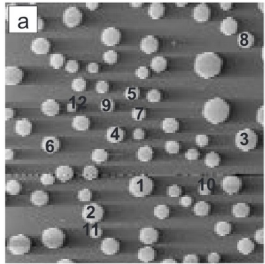
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Observation

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200 nm

Fig. 1a,b. STM images taken from a movie sequence showing Ostwald ripening of 0.21 ML Ag on Ag(111) at room temperature. The left image (a) was recorded 38 minutes after the completion of deposition, the right image (b) shows the same sample another 6 h and 56 min later. The area of the islands marked by numbers on the first image is plotted in Fig. 2 as a function of time. For better orientation, the islands 1 and 2 are marked on the right image as well

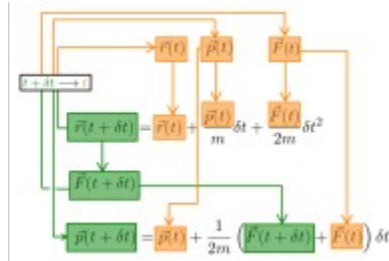
Rosenfeld et al, Appl Phys A 69, 489 (1999).

Mathematics: Taylor development
$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{d\mathbf{r}(t)}{dt} \delta t + \frac{1}{2} \frac{d^2\mathbf{r}(t)}{dt^2} \delta t^2 + \mathcal{O}(\delta t^3)$$

Physics: Newton equation
$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{\mathbf{p}(t)}{m} \delta t + \frac{1}{2} \frac{\mathbf{F}(t)}{m} \delta t^2 + \mathcal{O}(\delta t^3)$$

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Computational science: Velocity-Verlet algorithm



Why python?

- easier to use than C++ or FORTRAN

numpy.linalg.eigh

numpy.linalg.solve(a, b)

Lapack

```
/* Solve systems of linear equations  $Ax = b$  with the  
factored matrix, replacing b with solutions */  
dgbtrs('N', N, kl, ku, nrhs, A, lda, ipiv, b,  
ldb, &info);
```

Why python?

- easier to use than C++ or FORTRAN
- allows an object-oriented approach to programming

```
### Definition of the object "Atom"
class Atom:
    def __init__(self,r,p,mass=1):
        self.r=r
        self.p=p
        self.F=[0.0,0.0]
        self.Fold=[0.0,0.0]
        self.m=mass
        self.x=[r[0]]
        self.y=[r[1]]

### Method for moving atoms

    def move(self,dt,L):
        for i in range(2):
            # Verlet algorithm
            self.r[i]=self.r[i]+self.p[i]*dt/self.m+0.5*self.F[i]*(dt**2)/self.m
            # if the atom reaches the boundaries of the box
            if self.r[i] < 0.0:
                self.r[i]=-self.r[i]
                self.p[i]=-self.p[i]
            if self.r[i] > L:
                self.r[i]=2*L-self.r[i]
                self.p[i]=-self.p[i]
            self.x=numpy.append(self.x,self.r[0])
            self.y=numpy.append(self.y,self.r[1])

### Method for computing linear momentum of atoms
    def linear_momentum(self,dt):
        self.p[0]=self.p[0]+0.5*dt*(self.F[0]+self.Fold[0])
        self.p[1]=self.p[1]+0.5*dt*(self.F[1]+self.Fold[1])
```

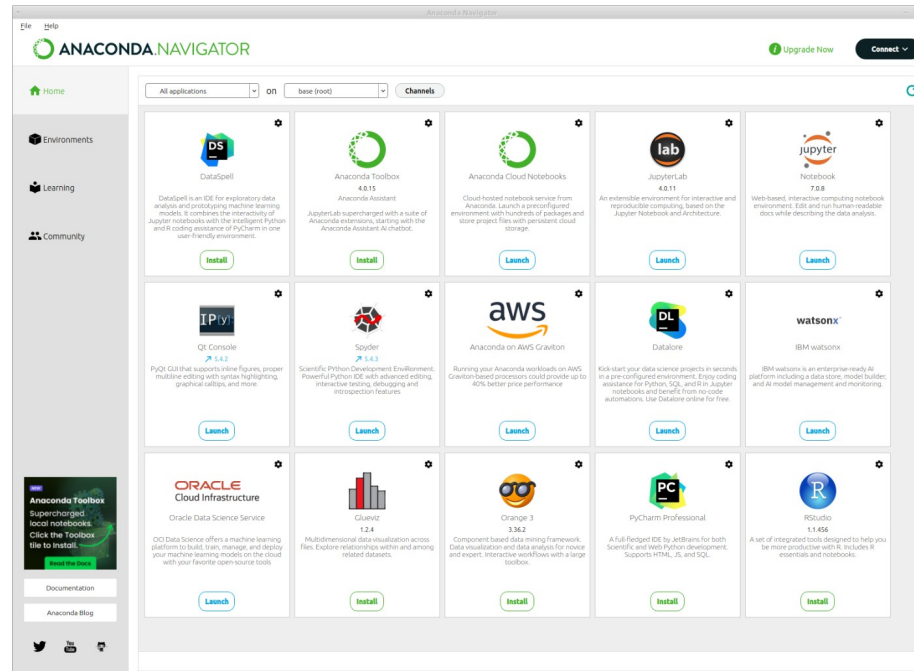
Why python?

- easier to use than C++ or FORTRAN
- allows an object-oriented approach to programming
- organization into modules

```
import numpy  
from matplotlib import pyplot
```

Why python?

- easier to use than C++ or FORTRAN
- allows an object-oriented approach to programming
- organization into modules
- a wide range of IDE (Integrated Development Environment) (spyder, jupyter, etc.)



Importing libraries

- to perform mathematical and scientific calculations

```
[1]: import numpy
```

- To draw curves

```
[2]: from matplotlib import pyplot
```

- to generate random numbers needed to initialize the velocities of the atoms

```
[3]: import random
```

Interatomic interaction

Lennard-Jones

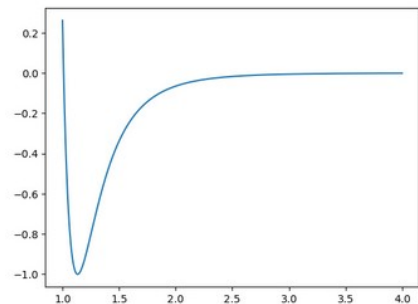
$$V_{LJ}(r_{ij}) = 4\epsilon \left(\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^6 \right)$$

Potential energy of the crystal:

$$E_{pot} = \sum_{i=1}^{N-1} \sum_{j=i+1}^N V_{LJ}(r_{ij})$$

```
[4]: eps=-1000.0
sig=0.25

sig=1.01
eps=1.0
def V(r,eps=sig,sig=sig):
    return 4*eps*( (sig/r)**12 - (sig/r)**6)
def graph(r,rx,eps=eps,sig=sig):
    return [-24*eps*rx*(2*(sig/r)**12 - (sig/r)**6)/r**2]
r=numpy.linspace(1.0,4.0,1000)
fig = pyplot.figure()
ax = fig.add_subplot()
pyplot.plot(r,V(r))
#ax.set_xlim(0.0,4.0)
#ax.set_ylim(-0.00001,0.00005)
#pyplot.legend()
pyplot.show()
```



Definition of objects

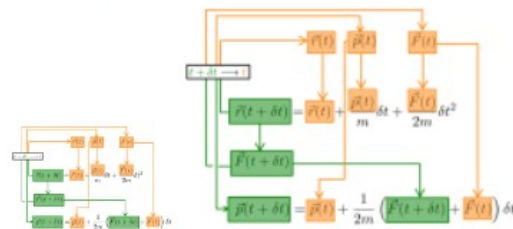
"Atom" object

The move() method moves an atom using a Velocity-Verlet algorithm.

$$\mathbf{r}(t + \delta t) = \mathbf{r}(t) + \frac{\mathbf{p}(t)}{m} \delta t + \frac{1}{2} \frac{\mathbf{F}(t)}{m} \delta t^2 + \mathcal{O}(\delta t^3)$$

$$\mathbf{p}(t + \delta t) = \mathbf{p}(t) + \frac{1}{2m} (\mathbf{F}(t + \delta t) + \mathbf{F}(t)) \delta t + \mathcal{O}(\delta t^3)$$

Velocity-Verlet algorithm



```
[0]: """ Definition of the object "Atom"
class Atom:
    def __init__(self,r,p,mass=1):
        self.r=r
        self.p=p
        self.F=[0.0,0.0]
        self.Fold=[0.0,0.0]
        self.m=mass
        self.x=[r[0]]
        self.y=[r[1]]

    """ Method for moving atoms

    def move(self,dt,L):
        for i in range(2):
            # Verlet algorithm
            self.r[i]=self.r[i]+self.p[i]*dt/self.m+0.5*self.F[i]*(dt**2)/self.m
            # if the atom reaches the boundaries of the box
            if self.r[i] < 0.0:
                self.r[i]=-self.r[i]
                self.p[i]=-self.p[i]
            if self.r[i] > L:
                self.r[i]=2*L-self.r[i]
                self.p[i]=-self.p[i]
            self.x=numpy.append(self.x,self.r[0])
            self.y=numpy.append(self.y,self.r[1])

    """ Method for computing linear momentum of atoms
    def linear_momentum(self,dt):
        self.p[0]=self.p[0]+0.5*dt*(self.F[0]+self.Fold[0])
        self.p[1]=self.p[1]+0.5*dt*(self.F[1]+self.Fold[1])
```

"Crystal" object

Definition of the object "Crystal"

```
[7]: class Crystal:
    def __init__(self):
        self.atoms=numpy.array([])
        self.Epot=0.0
        self.Ek=0.0

    # Method adding atoms into crystal

    def add_atom(self,r=numpy.array([0.0,0.0]),p=numpy.array([0.0,0.0]),mass=1.0):
        self.atoms=numpy.append(self.atoms,Atom(r,p,mass=mass))

    def rij(self,i,j):
        rx=self.atoms[j].r[0]-self.atoms[i].r[0]
        ry=self.atoms[j].r[1]-self.atoms[i].r[1]
        rij=numpy.sqrt(rx*rx+ry*ry)
        return rx,ry,rij

    def force(self):
        n_atoms=len(self.atoms)
        self.Epot=0.0
        self.Ek=0.0
        for atm in self.atoms:
            atm.Fold[0]=atm.F[0]
            atm.Fold[1]=atm.F[1]
            atm.F[0]=0.0
            atm.F[1]=0.0
        for i in range(n_atoms-1):
            for j in range(i+1,n_atoms):
                rx,ry,rij=self.rij(i,j)
                self.atoms[i].F[0]=self.atoms[i].F[0]+gradV(rij,rx)
                self.atoms[i].F[1]=self.atoms[i].F[1]+gradV(rij,ry)
                self.atoms[j].F[0]=self.atoms[j].F[0]-gradV(rij,rx)
                self.atoms[j].F[1]=self.atoms[j].F[1]-gradV(rij,ry)
                self.Epot=self.Epot+V(rij)
```

Building of a 2D "Crystal" of 5 atoms

```
[8]: L=10.0 # Size of the box in meter
m=00.0 # mass of the atoms in kg
velocity=1.0 # km/h # vitesse de 5 km/h <=> 1.4 m/s <=> p=83 kg.m/s
pini=m*(velocity*1e3/3600) # linear momentum
```

Positions of the atoms

```
[9]: positions=numpy.array([[L/2,L/2],[L/4,L/4],[3*L/4,L/4],[L/4,3*L/4],[3*L/4,3*L/4]])
```

Building the initial crystal "Ag"

```
[10]: Ag=Crystal()
```

```
[11]: for r in positions:
px=pini*(random.random()-.5)
py=pini*(random.random()-.5)
Ag.add_atom(r=r,p=[px,py],mass=m)
```

Computing initial forces

```
[12]: Ag.force()

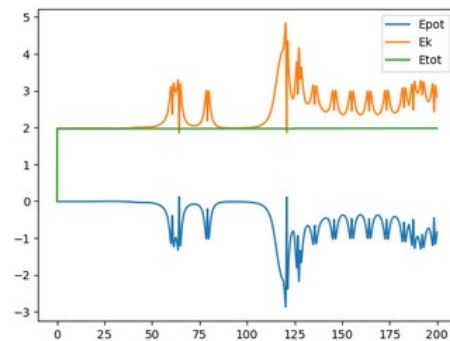
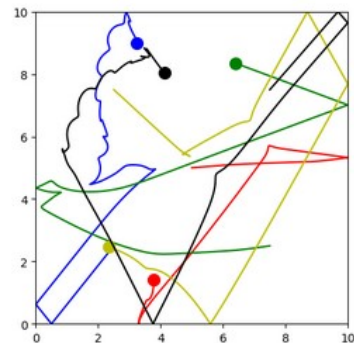
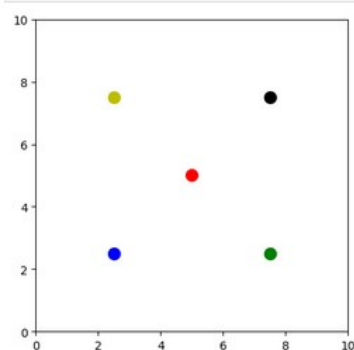
dt=.025
Epot=numpy.array([Ag.Epot])
Ek=numpy.array([Ag.Ek])
Etot=numpy.array([Ag.Epot+Ag.Ek])
temps=numpy.array([0.0])
```

Simulation of the movement of the atoms

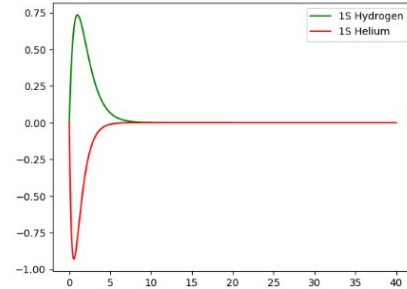
```
[13]: number_total_of_steps=9000
freq=number_total_of_steps-1
for nstep in range(number_total_of_steps):
    for atm in Ag.atoms:
        atm.move(dt,L)
        Ag.force()

    for atm in Ag.atoms:
        atm.linear_momentum(dt)
        Ag.Ek=Ag.Ek+0.5*(atm.p[0]**2+atm.p[1]**2)/atm.m
    Epot=numpy.append(Epot,Ag.Epot)
    Ek=numpy.append(Ek,Ag.Ek)
    Etot=numpy.append(Etot,Ag.Epot+Ag.Ek)
    temps=numpy.append(temps,(nstep+1)*dt)
    if nstep%freq == 0:
        display(Ag.atoms)

#display(Cu.atoms)
fig = pyplot.figure()
ax = fig.add_subplot()
pyplot.plot(temps,Epot,label="Epot")
pyplot.plot(temps,Ek,label="Ek")
pyplot.plot(temps,Etot,label="Etot")
pyplot.legend()
pyplot.show()
```



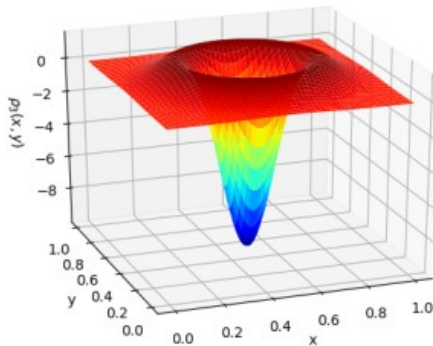
- Solving the one dimensional Schrödinger equation of the hydrogen and helium atom using the finite elements method



- Modelling of metallic nanoalloys at the atomic scale : chemical order/disorder transition
- Solving 2D Poisson equation by using Finite Difference Method

$$\nabla^2 \phi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\epsilon_0}$$

Source



Potential

